

# Scaling ansatz with texture zeros in linear seesaw

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## Abstract

We investigate scaling ansatz with texture zeros within the framework of linear seesaw mechanism. In this variant of seesaw mechanism a simplified expression of effective neutrino mass matrix  $m_\nu$  containing two Dirac type matrices ( $m_D$  and  $m_{DS}$ ) and one Majorana type matrix ( $m_{RS}$ ) is obtained by virtue of neglecting the global  $U(1)_L$  symmetry breaking term in the mass term of the Lagrangian. Along with the charged lepton mass matrix, the matrix  $m_{RS}$  too, is chosen in a diagonal basis whereas a scaling relation is incorporated in  $m_D$  and  $m_{DS}$  with different scale factors. Our goal in this work is to achieve a completely phenomenologically acceptable  $m_\nu$  generated by combinations of  $m_D$  and  $m_{DS}$  containing least number of independent parameters or maximum number of zeros. At the end of the numerical analysis it is found that number of zeros in any of the constituent Dirac type matrices ( $m_D$  and  $m_{DS}$ ) of  $m_\nu$  cannot be greater than six in order to meet the phenomenological requirements. The hierarchy obtained here is normal and also the values of the two parameters sum mass ( $\sum m_i$ ) and  $|m_{\nu ee}|$  are below the present experimental lower limit.

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# 1 Introduction

In the quest towards understanding of a viable flavour structure of low-energy neutrino mass matrix adhering neutrino oscillation data, a general approach is to advocate flavour symmetries in conjunction with the standard  $SU(2)_L \times U(1)_Y$  model. Those additional flavour symmetries are associated with some gauge group, discrete or continuous, and thereby dictating a well-defined theory to explain the extant data. This is a task to realize a comprehensive theory in the ultimate goal to comply with all experimental results. On the contrary, realization of a viable neutrino mass matrix through the proposition of some ansatz at low energy is also a supportive way towards the quest of a more elucidative model.

In the present work we investigate the latter idea considering two ansatzes, (i) zeros in the Yukawa texture, (ii) a scaling property between the nonzero Yukawa matrix elements, referred to as scaling ansatz[1]-[11] within the framework of a variant of seesaw mechanism known as “linear seesaw” mechanism[12]-[16]. We do not touch the origin of those two well-studied ansatzes here, however, we bring the two ansatzes together here and investigate systematically the minimal number of parameters necessary to explain the neutrino experimental data within the above mentioned framework. We briefly mention few words regarding scaling ansatz. Imposition of scaling ansatz correlates the nonzero elements of Yukawa matrix by a scale factor and it can be achieved through different ways. One of the distinctive properties of scaling ansatz is that the texture remains invariant under renormalization group evolution. Furthermore, this ansatz leads to  $m_3 = 0$  and  $\theta_{13} = 0$ . Thus we are compelled to break the ansatz to generate nonzero  $\theta_{13}$ .

Texture zeros[17]-[30] are investigated in the literature within different framework to generate light neutrino masses. Here, we start with maximum number of zeros in Yukawa matrix and investigate by reducing the number of zeros till we get a minimum of necessary parameters to explain neutrino oscillation data[31, 32, 33].

Our plan of the paper is as follows. Section 2 deals with linear seesaw mechanism framework. The scaling ansatz considered is given in section 3. Section 4 contains analysis with texture zeros. Parametrization and diagonalization of the emerged neutrino mass matrices is shown in Section 5. Discussion on numerical result is given in section 6. Section 7 contains the summary and conclusion of the present work.

## 2 Linear seesaw

In linear seesaw, the effective neutrino mass matrix ( $m_\nu$ ) generated varies linearly with Dirac neutrino mass matrix ( $m_D$ ) instead of quadratic variation as happens in type-I seesaw. In this popular variant of type-I seesaw along with left chiral SM doublet neutrinos ( $\nu_L$ ) and right chiral singlet neutrinos ( $N_{iR}$ ), extra fermion singlets ( $S_{iR}$ ) are added. In effect the well-known type-I seesaw basis ( $(\nu^c)_R, N_R$ ) is extended to  $((\nu^c)_R, N_R, S_R)$ . Linear seesaw mechanism arises when the mass matrix in the above basis takes the following form

$$M_\nu = \begin{pmatrix} 0 & m_D & m_{DS} \\ m_D^T & 0 & m_{RS} \\ m_{DS}^T & m_{RS}^T & M_S \end{pmatrix} \quad (2.1)$$

where  $M_\nu$  is a  $9 \times 9$  matrix assuming three generations of each fermions. To obtain light neutrino masses, we have to block diagonalize (2.1) and with the introduction of the following matrices

$$\begin{aligned} M_D &= \begin{pmatrix} m_D & m_{DS} \end{pmatrix}, \\ M_R &= \begin{pmatrix} 0 & m_{RS} \\ m_{RS}^T & M_S \end{pmatrix} \end{aligned} \quad (2.2)$$

the effective  $M_\nu$  takes the form as

$$M_\nu = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix} \quad (2.3)$$

which is exactly similar to that of type-I seesaw mass matrix. Further assuming the hierarchy of type-I seesaw mechanism the light neutrino mass matrix is obtained easily as

$$\begin{aligned} m_\nu &= -M_D M_R^{-1} M_D^T \\ &= m_D (m_{RS}^T)^{-1} M_S (m_{RS})^{-1} m_{DS}^T - m_D (m_{RS}^T)^{-1} m_{DS}^T - m_{DS} m_{RS}^{-1} m_D^T. \end{aligned} \quad (2.4)$$

If the  $U(1)_L$  global lepton number symmetry breaking term  $M_S$  is absent, (2.4) is then simply reduced to

$$m_\nu = -m_D (m_{RS}^T)^{-1} m_{DS}^T - m_{DS} m_{RS}^{-1} m_D^T. \quad (2.5)$$

This is our main working formula for the present work and we proceed further to calculate light neutrino masses and mixing angles with this  $m_\nu$  imposing scaling ansatz and texture zeros on the mass matrices  $m_D$  and  $m_{DS}$ . Moreover, without any loss of generality, we assume that the charged lepton mass matrix and  $m_{RS}$  are flavour diagonal. With such choice of basis it is not possible to consider further any other matrix flavour diagonal.

### 3 Scaling ansatz

There are several works[1]-[9] in which the scaling ansatz has been studied through its imposition on the columns of  $m_\nu$ . In the present work we consider scaling ansatz at a more fundamental level through its implementation in  $m_D$  and  $m_{DS}$ . Furthermore, we impose this ansatz along the rows of the  $m_D$  and  $m_{DS}$  matrices and we find that such choice of  $m_D$  and  $m_{DS}$  leads to the same structure of  $m_\nu$ , after invoking linear seesaw mechanism. Scaling ansatz dictates that all the elements of a certain row of  $m_D$  (or  $m_{DS}$ ) are related to the elements of another row by a definite scale factor. This scaling relation may be of three types as (i) the first and second row are connected, (ii) the second and third row are connected or (iii) the first and third row are connected. The cases (i) and (iii) lead to  $\theta_{12}$  or  $\theta_{23}$  equals to zero and hence, we discard those cases. Here we carry out our analysis for the case (ii) which is explicitly written as

$$\begin{aligned}(m_D)_{\mu i} &= k(m_D)_{\tau i} \\ (m_{DS})_{\mu i} &= k_1(m_{DS})_{\tau i},\end{aligned}\tag{3.1}$$

where  $i$  is the column index ( $i = 1, 2, 3$ ) and  $k, k_1$  are the scale factors for  $m_D$  and  $m_{DS}$  respectively. We now check the effect of the scaling ansatz in the effective neutrino mass matrix  $m_\nu$ . Using the linear seesaw formula (2.5) we obtain  $m_\nu$  as

$$\begin{aligned}(m_\nu)_{\mu\alpha} &= -[(m_D)_{\mu j}(m_{RS}^T)_{jl}^{-1}(m_{DS})_{l\alpha}^T + (m_{DS})_{\mu j}(m_{RS})_{jl}^{-1}(m_D)_{l\alpha}^T] \\ &= -[k(m_D)_{\tau j}(m_{RS}^T)_{jl}^{-1}(m_{DS})_{l\alpha}^T + k_1(m_{DS})_{\tau j}(m_{RS})_{jl}^{-1}(m_D)_{l\alpha}^T]\end{aligned}\tag{3.2}$$

where sum over repeated index is implied. It is clear from the above equation that the scaling ansatz is already broken by the choice of different scale factors for  $m_D$  and  $m_{DS}$ . The ansatz can be restored in  $m_\nu$  simply by choosing  $k = k_1$  and then (3.2) becomes

$$(m_\nu)_{\mu\alpha} = k(m_\nu)_{\tau\alpha}\tag{3.3}$$

with  $\alpha = e, \mu, \tau$  and the scaling relations in  $m_\nu$  are obtained as

$$\frac{(m_\nu)_{\mu e}}{(m_\nu)_{\tau e}} = \frac{(m_\nu)_{\mu\mu}}{(m_\nu)_{\tau\mu}} = \frac{(m_\nu)_{\mu\tau}}{(m_\nu)_{\tau\tau}} = k.\tag{3.4}$$

As we are aware of the fact that such type of scaling ansatz invariant matrices yield  $\theta_{13} = 0$ , we are compelled to deal with the case where  $k \neq k_1$ . The explicit forms of  $m_D$  and  $m_{DS}$  with scaling ansatz are given by

$$\begin{aligned}m_D &= \begin{pmatrix} a_1 & a_2 & a_3 \\ kb_1 & kb_2 & kb_3 \\ b_1 & b_2 & b_3 \end{pmatrix}, \\ m_{DS} &= \begin{pmatrix} x_1 & x_2 & x_3 \\ k_1 y_1 & k_1 y_2 & k_1 y_3 \\ y_1 & y_2 & y_3 \end{pmatrix}\end{aligned}\tag{3.5}$$

and as mentioned earlier  $m_{RS}$  is taken diagonal as

$$m_{RS} = \text{diag}(m_1, m_2, m_3). \quad (3.6)$$

## 4 Texture zeros

In our scheme developed we further put constraint on  $m_D$  and  $m_{DS}$  through the imposition of zeros and our aim here is to find out the maximum number of zeros that we can accommodate in  $m_D$  and  $m_{DS}$  which will produce a phenomenologically viable  $m_\nu$ . We start our analysis with 8 zero texture, and then move on by reducing the number of zeros. We calculate  $m_\nu$  for all possible combinations of  $m_D$  and  $m_{DS}$  and check how many of them can give rise to nonzero mixing angles and mass squared differences and how many can be ruled out at once using suitable arguments. First we tabulate scaling ansatz invariant  $n$  zero (where  $n = 8, 7, 6, \dots$ ) textures of  $m_D$  (and  $m_{DS}$ ). In Table 1 we represent the scaling ansatz invariant texture zero structures of  $m_D$  matrices. First of all  $m_D$  with 8 zeros is not possible because scaling ansatz requires at least two nonzero elements (one in each row connected by scaling). Texture with 7 zeros and 6 zeros are allowed due to compatibility with scaling ansatz. A table completely identical to Table 1 can be constructed for  $m_{DS}$  matrix simply by the following substitutions:  $a_i \rightarrow x_i$ ,  $b_i \rightarrow y_i$  and  $k \rightarrow k_1$  (where  $i = 1, 2, 3$ ). Thus there are three 7 zero and nine 6 zero textures allowed for both  $m_D$  and  $m_{DS}$ . Now we calculate  $m_\nu$  using linear seesaw formula (3.2) for all possible combinations of  $m_D$  and  $m_{DS}$  and there are altogether 144 different possible combinations. Depending upon the position of zeros in the resulting  $m_\nu$  matrices we divide those 144 textures in 8 classes and denote them as

$$\begin{aligned} t_1 &= \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}, t_2 = \begin{pmatrix} 0 & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}, t_3 = \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & \times \\ 0 & \times & \times \end{pmatrix}, t_4 = \begin{pmatrix} \times & \times & \times \\ \times & 0 & 0 \\ \times & 0 & 0 \end{pmatrix}, \\ t_5 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & \times & \times \\ 0 & \times & \times \end{pmatrix}, t_6 = \begin{pmatrix} 0 & \times & \times \\ \times & 0 & 0 \\ \times & 0 & 0 \end{pmatrix}, t_7 = \begin{pmatrix} \times & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, t_8 = \text{null matrix} \end{aligned} \quad (4.1)$$

where  $\times$  denotes generically some nonzero element. Exact expression of  $\times$  comes out from the structure of corresponding  $m_D$  and  $m_{DS}$ . Explicitly emergence of all those classified forms are shown in Tables 2-4. The tables of  $m_\nu$  (Tables 2-4) are presented in a matrix form in which along the row we write the index of  $m_D$  (denoted as  $i$ ) and along the columns the index of  $m_{DS}$  (denoted as  $j$ ) is assigned. Hence the  $ij$ th element of the table denotes the type of  $m_\nu$  generated by the combination of  $m_D(i)$  and  $m_{DS}(j)$ . We mark the surviving textures by bold letters in those tables.

Lets first check how many of these  $t_i$  matrices have the potential to generate phenomenologically viable mixing angles and mass eigenvalues. It has been shown by Frampton et.al[17] that if the

Table 1: List of  $n$  zero  $m_D$  matrices ( $n = 8, 7, 6$ )

8 zero texture		
No allowed texture		
7 zero texture		
$a_1 = a_2 = a_3 = 0$ and $b_2 = b_3 = 0$ $m_D(1) = \begin{pmatrix} 0 & 0 & 0 \\ kb_1 & 0 & 0 \\ b_1 & 0 & 0 \end{pmatrix}$	$a_1 = a_2 = a_3 = 0$ and $b_1 = b_3 = 0$ $m_D(2) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & kb_2 & 0 \\ 0 & b_2 & 0 \end{pmatrix}$	$a_1 = a_2 = a_3 = 0$ and $b_1 = b_2 = 0$ $m_D(3) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & kb_3 \\ 0 & 0 & b_3 \end{pmatrix}$
6 zero texture		
$a_2 = a_3 = 0$ and $b_2 = b_3 = 0$ $m_D(1) = \begin{pmatrix} a_1 & 0 & 0 \\ kb_1 & 0 & 0 \\ b_1 & 0 & 0 \end{pmatrix}$	$a_1 = a_3 = 0$ and $b_2 = b_3 = 0$ $m_D(2) = \begin{pmatrix} 0 & a_2 & 0 \\ kb_1 & 0 & 0 \\ b_1 & 0 & 0 \end{pmatrix}$	$a_1 = a_2 = 0$ and $b_2 = b_3 = 0$ $m_D(3) = \begin{pmatrix} 0 & 0 & a_3 \\ kb_1 & 0 & 0 \\ b_1 & 0 & 0 \end{pmatrix}$
$a_1 = a_3 = 0$ and $b_1 = b_3 = 0$ $m_D(4) = \begin{pmatrix} 0 & a_2 & 0 \\ 0 & kb_2 & 0 \\ 0 & b_2 & 0 \end{pmatrix}$	$a_2 = a_3 = 0$ and $b_1 = b_3 = 0$ $m_D(5) = \begin{pmatrix} a_1 & 0 & 0 \\ 0 & kb_2 & 0 \\ 0 & b_2 & 0 \end{pmatrix}$	$a_1 = a_2 = 0$ and $b_1 = b_3 = 0$ $m_D(6) = \begin{pmatrix} 0 & 0 & a_3 \\ 0 & kb_2 & 0 \\ 0 & b_2 & 0 \end{pmatrix}$
$a_1 = a_2 = 0$ and $b_1 = b_2 = 0$ $m_D(7) = \begin{pmatrix} 0 & 0 & a_3 \\ 0 & 0 & kb_3 \\ 0 & 0 & b_3 \end{pmatrix}$	$a_2 = a_3 = 0$ and $b_1 = b_2 = 0$ $m_D(8) = \begin{pmatrix} a_1 & 0 & 0 \\ 0 & 0 & kb_3 \\ 0 & 0 & b_3 \end{pmatrix}$	$a_1 = a_3 = 0$ and $b_1 = b_2 = 0$ $m_D(9) = \begin{pmatrix} 0 & a_2 & 0 \\ 0 & 0 & kb_3 \\ 0 & 0 & b_3 \end{pmatrix}$

number of independent zeros in an effective neutrino mass matrix ( $m_\nu$ ) is  $\geq 3$ , that matrix doesn't favour the oscillation data. This result drastically eliminates the matrices from  $t_4$  to  $t_8$ . Thus the matrices given in given in Table 2 (containing 7 zero  $m_D$  and 7 zero  $m_{DS}$ ) are all ruled out. Again although  $t_3$  matrix survives this criteria ( since the number of independent zeros in  $t_3$  matrix is only 2), however, one generation of neutrino is completely decoupled from the other two, and as a result two mixing angles become zero. Hence we also neglect  $t_3$  matrix. Thus the number of surviving  $t_i$  matrix is only 2 and they are  $t_1$  and  $t_2$ . The matrix  $t_1$  appears only in Table 5 due to three different combinations of  $m_D$  and  $m_{DS}$  (6 zero  $m_D$  with 6 zero  $m_{DS}$ ) and matrix  $t_2$  appears in Tables 3-5 due to total 18 different combinations of the above matrices. Both Tables 3 (combination of 6 zero  $m_D$  and 7 zero  $m_{DS}$ ) and 4 (7 zero  $m_D$  and 6 zero  $m_{DS}$ ) give three  $t_2$  matrices each and Table 5

gives three  $t_1$  matrices and twelve  $t_2$  matrices.

Table 2: Combination of 7 zero  $m_D(i)$  and 7 zero  $m_{DS}(j)$

		Type of $m_\nu$		
$j$	$i$	1	2	3
1		$t_5$	$t_8$	$t_8$
2		$t_8$	$t_5$	$t_8$
3		$t_8$	$t_8$	$t_5$

Table 3: Combination of 6 zero  $m_D(i)$  and 7 zero  $m_{DS}(j)$

		Type of $m_\nu$								
$j \backslash i$		1	2	3	4	5	6	7	8	9
1		$\boldsymbol{t_2}$	$t_5$	$t_5$	$t_8$	$t_6$	$t_8$	$t_8$	$t_6$	$t_8$
2		$t_8$	$t_6$	$t_8$	$\boldsymbol{t_2}$	$t_5$	$t_6$	$t_8$	$t_8$	$t_6$
3		$t_8$	$t_8$	$t_6$	$t_8$	$t_8$	$t_6$	$\boldsymbol{t_2}$	$t_5$	$t_5$

Table 4: Combination of 7 zero  $m_D(i)$  and 6 zero  $m_{DS}(j)$

		Type of $m_\nu$		
$j$	$i$	1	2	3
1		$\mathbf{t}_2$	$t_8$	$t_8$
2		$t_5$	$t_6$	$t_8$
3		$t_5$	$t_8$	$t_6$
4		$t_8$	$\mathbf{t}_2$	$t_8$
5		$t_6$	$t_5$	$t_8$
6		$t_8$	$t_5$	$t_6$
7		$t_8$	$t_8$	$\mathbf{t}_2$
8		$t_6$	$t_8$	$t_5$
9		$t_8$	$t_6$	$t_5$

Table 5: Combination of 6 zero  $m_D(i)$  and 6 zero  $m_{DS}(j)$

		Type of $m_\nu$								
$j \backslash i$	$i$	1	2	3	4	5	6	7	8	9
1		$\mathbf{t}_1$	$\mathbf{t}_2$	$\mathbf{t}_2$	$t_8$	$t_4$	$t_8$	$t_8$	$t_4$	$t_8$
2		$\mathbf{t}_2$	$t_3$	$t_5$	$t_4$	$t_6$	$t_6$	$t_8$	$t_6$	$t_7$
3		$\mathbf{t}_2$	$t_5$	$t_3$	$t_8$	$t_6$	$t_7$	$t_4$	$t_6$	$t_6$
4		$t_8$	$t_4$	$t_8$	$\mathbf{t}_1$	$\mathbf{t}_2$	$\mathbf{t}_2$	$t_8$	$t_8$	$t_4$
5		$t_4$	$t_6$	$t_6$	$\mathbf{t}_2$	$t_3$	$t_5$	$t_8$	$t_7$	$t_6$
6		$t_8$	$t_6$	$t_7$	$\mathbf{t}_2$	$t_5$	$t_3$	$t_4$	$t_6$	$t_6$
7		$t_8$	$t_8$	$t_4$	$t_8$	$t_8$	$t_4$	$\mathbf{t}_1$	$\mathbf{t}_2$	$\mathbf{t}_2$
8		$t_4$	$t_8$	$t_6$	$t_8$	$t_7$	$t_6$	$\mathbf{t}_2$	$t_3$	$t_5$
9		$t_8$	$t_7$	$t_6$	$t_4$	$t_6$	$t_6$	$\mathbf{t}_2$	$t_5$	$t_3$

Table 2 shows that none of the 9 combinations survives. Both Table 3 and 4 give us 3 viable  $m_\nu$ s where as Table 5 gives 15 surviving combinations (including both  $\mathbf{t}_1$  and  $\mathbf{t}_2$ ).

## 5 Parametrization and diagonalization

In this section at first we write down the explicit forms of the surviving  $m_\nu$  s in terms of model parameters and again parametrize them in a convenient way. Lets start with Table 3 considering the combination of 6 zero  $m_D$  and 7 zero  $m_{DS}$ . For ( $i = 1, j = 1$ )

$$m_\nu = -\frac{1}{m_1} \begin{pmatrix} 0 & k_1 a_1 y_1 & a_1 y_1 \\ k_1 a_1 y_1 & 2k k_1 b_1 y_1 & (k + k_1) b_1 y_1 \\ a_1 y_1 & (k + k_1) b_1 y_1 & 2b_1 y_1 \end{pmatrix}. \quad (5.1)$$

We assume that the scale factors  $k$  and  $k_1$  are related by a breaking parameter  $\epsilon$  as  $k_1 = k(1 + \epsilon)$  such that  $k$  and  $k_1$  becomes equal when  $\epsilon$  vanishes. The above matrix (5.1) after the substitutions

$$qe^{i\beta} = \frac{a_1 y_1}{m_1}, \quad te^{i\gamma} = \frac{b_1 y_1}{m_1} \quad (5.2)$$

becomes

$$m_\nu = \begin{pmatrix} 0 & kq(1 + \epsilon) & q \\ kq(1 + \epsilon) & 2k^2 te^{i\theta_2}(1 + \epsilon) & 2kte^{i\theta_2} + \epsilon kte^{i\theta_2} \\ q & 2kte^{i\theta_2} + \epsilon kte^{i\theta_2} & 2te^{i\theta_2} \end{pmatrix} \quad (5.3)$$



where we have taken out the phase  $\beta$  and the negative sign by the rotation

$$m_\nu \rightarrow e^{-\frac{i\pi}{2}} \text{diag}(1 \ e^{-i\beta} \ e^{-i\beta}) m_\nu e^{-\frac{i\pi}{2}} \text{diag}(1 \ e^{-i\beta} \ e^{-i\beta}) \quad (5.4)$$

and renamed the existing phase as  $\theta_2 = \gamma - 2\beta$ . Lets denote the above mass matrix as **Cat I**. Effective neutrino mass matrix ( $m_\nu$ ) for other two surviving combinations, ( $i = 4, j = 2$ ) and ( $i = 7, j = 3$ ) looks identical to (5.3) but with different parametrizations as

$$qe^{i\beta} = \frac{a_2 y_2}{m_2}, \quad te^{i\gamma} = \frac{b_2 y_2}{m_2} \quad (5.5)$$

and

$$qe^{i\beta} = \frac{a_3 y_3}{m_3}, \quad te^{i\gamma} = \frac{b_3 y_3}{m_3} \quad (5.6)$$

respectively. It is to be noted that we have dubbed the parameters  $p, q, \beta, \gamma$  for rest of the cases.

Moving on to the next set of combinations i.e Table 4 (7 zero  $m_D$  and 6 zero  $m_{DS}$ ), we find that for ( $i = 1, j = 1$ )

$$m_\nu = -\frac{1}{m_1} \begin{pmatrix} 0 & kb_1 x_1 & b_1 x_1 \\ kb_1 x_1 & 2kk_1 b_1 y_1 & (k + k_1) b_1 y_1 \\ b_1 x_1 & (k + k_1) b_1 y_1 & 2b_1 y_1 \end{pmatrix} \quad (5.7)$$

which after the combined substitutions

$$qe^{i\beta} = \frac{b_1 x_1}{m_1}, \quad te^{i\gamma} = \frac{b_1 y_1}{m_1} \quad (5.8)$$

and rotation

$$m_\nu \rightarrow e^{-\frac{i\pi}{2}} \text{diag}(1 \ e^{-i\beta} \ e^{-i\beta}) m_\nu e^{-\frac{i\pi}{2}} \text{diag}(1 \ e^{-i\beta} \ e^{-i\beta}) \quad (5.9)$$

becomes

$$m_\nu = \begin{pmatrix} 0 & kq & q \\ kq & 2k^2 te^{i\theta_2} (1 + \epsilon) & 2kte^{i\theta_2} + \epsilon kte^{i\theta_2} \\ q & 2kte^{i\theta_2} + \epsilon kte^{i\theta_2} & 2te^{i\theta_2} \end{pmatrix} \quad (5.10)$$

where  $\theta_2 = \gamma - 2\beta$ . The resulting neutrino mass matrix (5.10) is named as **Cat II**.

The  $m_\nu$  appeared for other two surviving combinations ( $i = 2, j = 4$ ) and ( $i = 3, j = 7$ ) are written in a generic way through (5.10) with the following choices of parameters as

$$qe^{i\beta} = \frac{b_2 x_2}{m_2}, \quad te^{i\gamma} = \frac{b_2 y_2}{m_2} \quad (5.11)$$

$$qe^{i\beta} = \frac{b_3 x_3}{m_3}, \quad te^{i\gamma} = \frac{b_3 y_3}{m_3} \quad (5.12)$$

respectively. Both the **Cat I** and **Cat II** matrices have 11 th element zero indicating the fact that they have vanishing  $|m_{\nu_{ee}}|$ . Unlike the previous two sets of combinations, the third one shown in

Table 5 ( 6 zero  $m_D$  and 6 zero  $m_{DS}$  ) possess some combinations for which the resulting  $m_\nu$  have all its elements nonzero (denoted by  $\mathbf{t}_1$  ). Table 5 shows that the combination of 6 zero  $m_D$  and 6 zero  $m_{DS}$  gives rise to a total of 15 viable structure of  $m_\nu$  s of which 3 belongs to  $\mathbf{t}_1$  type and remaining 12 are of  $\mathbf{t}_2$  type. We start with  $(i = 1, j = 1)$  for which explicit form of  $m_\nu$  is given by

$$m_\nu = -\frac{1}{m_1} \begin{pmatrix} 2a_1x_1 & kb_1x_1 + k_1a_1y_1 & b_1x_1 + a_1y_1 \\ kb_1x_1 + k_1a_1y_1 & 2kk_1b_1y_1 & (k + k_1)b_1y_1 \\ b_1x_1 + a_1y_1 & (k + k_1)b_1y_1 & 2b_1y_1 \end{pmatrix}. \quad (5.13)$$

where  $k_1 = k(1 + \epsilon)$ . To get a convenient form of  $m_\nu$  we use the parametrizations

$$pe^{i\alpha} = \frac{2a_1x_1}{m_1}, \quad qe^{i\beta} = \frac{b_1x_1 + a_1y_1}{m_1}, \quad q_1e^{i\beta_1} = \frac{a_1y_1}{m_1}, \quad te^{i\gamma} = \frac{b_1y_1}{m_1}. \quad (5.14)$$

At first we rewrite the above matrix using these substitutions and then take out the redundant phases using the rotation

$$m_\nu \rightarrow e^{-\frac{i\pi}{2}} \text{diag}(e^{-i\frac{\alpha}{2}} \ e^{-i\beta} \ e^{-i\beta}) m_\nu e^{-\frac{i\pi}{2}} \text{diag}(e^{-i\frac{\alpha}{2}} \ e^{-i\beta} \ e^{-i\beta}). \quad (5.15)$$

Finally we arrive at a suitable form of  $m_\nu$  as

$$m_\nu = \begin{pmatrix} p & kq + \epsilon kq_1e^{i\theta_1} & q \\ kq + \epsilon kq_1e^{i\theta_1} & 2k^2te^{i\theta_2}(1 + \epsilon) & 2kte^{i\theta_2} + \epsilon kte^{i\theta_2} \\ q & 2kte^{i\theta_2} + \epsilon kte^{i\theta_2} & 2te^{i\theta_2} \end{pmatrix} \quad (5.16)$$

where the remaining phases are redefined as  $\theta_1 = \beta_1 - \beta$  and  $\theta_2 = \gamma - 2\beta$ . This structure of  $m_\nu$  (5.16) is designated as **Cat III**. The other two  $\mathbf{t}_1$  type  $m_\nu$  s can be brought into this form with some different parametrizations given by

$$pe^{i\alpha} = \frac{2a_2x_2}{m_2}, \quad qe^{i\beta} = \frac{b_2x_2 + a_2y_2}{m_2}, \quad q_1e^{i\beta_1} = \frac{a_2y_2}{m_2}, \quad te^{i\gamma} = \frac{b_2y_2}{m_2} \quad (5.17)$$

for  $(i = 4, j = 4)$  and

$$pe^{i\alpha} = \frac{2a_3x_3}{m_3}, \quad qe^{i\beta} = \frac{b_3x_3 + a_3y_3}{m_3}, \quad q_1e^{i\beta_1} = \frac{a_3y_3}{m_3}, \quad te^{i\gamma} = \frac{b_3y_3}{m_3} \quad (5.18)$$

for  $(i = 7, j = 7)$ . We can recast all the remaining  $\mathbf{t}_2$  type  $m_\nu$  to either **Cat I** or **Cat II** and required parametrizations are given below. Among these 12 structures, only 6 are different from each other, i.e we get 6 pairs and in each pair one is completely identical to the other. We denote these pairs in a second bracket as (i){ $(i = 1, j = 2)$  and  $(i = 1, j = 3)$ }, (ii){ $(i = 4, j = 5)$  and  $(i = 4, j = 6)$ }, (iii){ $(i = 7, j = 8)$  and  $(i = 7, j = 9)$ }, (iv){ $(i = 2, j = 1)$  and  $(i = 3, j = 1)$ }, (v){ $(i = 5, j = 4)$  and  $(i = 6, j = 4)$ }, (vi){ $(i = 8, j = 7)$  and  $(i = 9, j = 7)$ }. The first three

pairs (i), (ii) and (iii) can be expressed by the generic matrix of **Cat I** with parametrizations

$$\begin{aligned} qe^{i\beta} &= \frac{a_1 y_1}{m_1}, \quad te^{i\gamma} = \frac{b_1 y_1}{m_1} \\ qe^{i\beta} &= \frac{a_2 y_2}{m_2}, \quad te^{i\gamma} = \frac{b_2 y_2}{m_2} \\ qe^{i\beta} &= \frac{a_3 y_3}{m_3}, \quad te^{i\gamma} = \frac{b_3 y_3}{m_3} \end{aligned} \quad (5.19)$$

respectively, whereas the last three pairs (iv), (v) and (vi) produce that of **Cat II** and the required parametrizations are

$$\begin{aligned} qe^{i\beta} &= \frac{b_1 x_1}{m_1}, \quad te^{i\gamma} = \frac{b_1 y_1}{m_1} \\ qe^{i\beta} &= \frac{b_2 x_2}{m_2}, \quad te^{i\gamma} = \frac{b_2 y_2}{m_2} \\ qe^{i\beta} &= \frac{b_3 x_3}{m_3}, \quad te^{i\gamma} = \frac{b_3 y_3}{m_3}. \end{aligned} \quad (5.20)$$

It is clear from the above analysis that all the viable (a total of 21)  $m_\nu$  matrices can be written in three categories namely **Cat I**, **Cat II** and **Cat III** after parametrization. So it is enough to analyze only these three matrices numerically to examine whether they have any allowed parameter space.

## 6 Discussion on numerical results

Now our task is to obtain the exact values of the neutrino oscillation observables (mass squared differences and mixing angles) of the surviving  $m_\nu$  matrices belonging to **Cat I**, **Cat II** and **Cat III**. We use straightforward generalized diagonalization methodology developed earlier[34] to calculate mass eigenvalues, mixing angles and CP violating phases - both Dirac and Majorana type in terms of the mass matrix parameters. Neutrino oscillation experimental data generated from global fit (Table 6) is used to obtain the admissible parameter space. In this work the experimental constraints used to restrict the parameters are solar and atmospheric mass squared differences and three mixing angles and we predict the individual neutrino masses, the corresponding hierarchy, their sum ( $\sum m_i$ ), the value of  $|m_{\nu ee}|$ , the CP violating Jarlskog invariant  $J_{CP}$  and the Dirac CP violating phase  $\delta_D$ . We also predict the value of the Majorana phases, which will be tested [35, 36] in neutrinoless double beta decay experiments, however, determination of their values is a challenging task.

First we analyze the **Cat I** and **Cat II** matrices where we encounter the vanishing  $|m_{\nu ee}|$  element. Although the explicit structure of these two matrices are different from each other they

Table 6: Input data from neutrino oscillation experiments [33]

Quantity	$3\sigma$ ranges/other constraint
$\Delta m_{21}^2$	$7.12 < \Delta m_{21}^2 (10^5 \text{ eV}^{-2}) < 8.20$
$ \Delta m_{31}^2 (N)$	$2.31 < \Delta m_{31}^2 (10^3 \text{ eV}^{-2}) < 2.74$
$ \Delta m_{31}^2 (I)$	$2.21 < \Delta m_{31}^2 (10^3 \text{ eV}^{-2}) < 2.64$
$\theta_{12}$	$31.30^\circ < \theta_{12} < 37.46^\circ$
$\theta_{23}$	$36.86^\circ < \theta_{23} < 55.55^\circ$
$\theta_{13}$	$7.49^\circ < \theta_{13} < 10.46^\circ$

are composed of same 5 parameters namely  $k$ ,  $q$ ,  $t$ ,  $\epsilon$  and a phase parameter  $\theta_2$ . After scanning those parameters in various possible ranges we find that both of them (**Cat I** and **Cat II**) fail to produce all the neutrino oscillation observables simultaneously inside the allowed  $3\sigma$  range as mentioned above (Table 6). It has been observed that both of the above matrices can produce all the experimental observables except  $\theta_{13}$  inside the allowed range. The lowest  $\theta_{13}$  produced here exceeds the upper limit of the  $3\sigma$  range quoted in Table 6. Hence, the  $m_\nu$  matrices grouped in **Cat I** and **Cat II** are discarded. We are now left with only one type of  $m_\nu$  (**Cat III**). The matrix belonging to **Cat III** is made up of total 8 parameters and they are  $p$ ,  $q$ ,  $k$ ,  $t$ ,  $q_1$ ,  $\epsilon$  and two phase parameters  $\theta_1$ ,  $\theta_2$ . Varying those parameters in nearly all possible ranges we find some admissible parameter space satisfying extant data. Ranges of the allowed parameters for which the values of the resulting oscillation observables fall within  $3\sigma$  range of extant data are shown in the Table 7 below. The phase parameter space is divided in four patches in  $\theta_1$  vs  $\theta_2$  plane and pairwise one is

Table 7: Allowed ranges of parameters

Parameters	$p$	$q$	$k$	$t$	$q_1$	$\epsilon$
<b>Allowed ranges</b>	0.001-0.016	0.001-0.053	0.2-0.9	0.006-0.028	0.01-0.1	1.4-9.5

mirror image to the other. The allowed values of  $\theta_1$  and  $\theta_2$  are shown in Table 8. In Table 9 we predict the individual mass eigenvalues and the sum of the three neutrino masses ( $\sum m_i$ ) and the value of  $|m_{\nu_{ee}}|$ .

Some comments on the issue of the predictions of the present scheme are in order.

1. First of all the mass ordering obtained in the present scheme is normal and it is illustrated in the left panel of Fig.1 through a plot of  $m_1$  with  $m_2$  and  $m_3$ . It has been shown[37, 38, 39]

Table 8: Allowed ranges of  $\theta_1$  and  $\theta_2$  phase parameters

$\theta_1$ (deg.)	$\theta_2$ (deg.)
$(-180)-(-73.2)$	$(-35)-(-180)$
and	
$(-112.3)-(-35.5)$	$(-180)-(-54.3)$

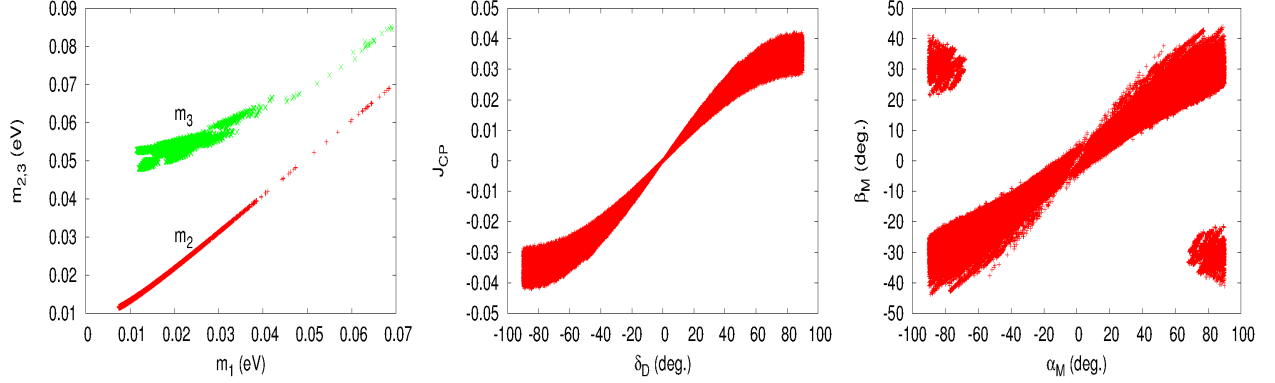


Figure 1: (colour online) Plot of mass eigenvalues  $m_1$  vs  $m_{2,3}$  (left), Jarlskog measure( $J_{CP}$ ) vs Dirac CP Phase( $\delta_D$ ) (middle) and Majorana Phases ( $\alpha_M$  vs  $\beta_M$ ) (right)

that precise determination of  $\theta_{13}$  through reactor neutrino experiments will enable us to fix the neutrino mass ordering through a combined analysis complying with the results of long baseline experiments NO $\nu$ A[40, 41] and T2K[42], since, result of only one of them is insufficient to probe the mass ordering due to degenerate nature of  $\delta_{CP}$  in the expression of  $P(\nu_\mu \rightarrow \nu_e)$ [37, 43]. Thus the prediction of the hierarchy of the present scheme will be tested in near future.

Next we have plotted  $J_{CP}$  with  $\delta_D$  in the middle panel of Fig.1. Information on the value of  $J_{CP}$  can be obtained from the experiment looking for the difference between  $P(\nu_\mu \rightarrow \nu_e)$  and  $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$  using neutrino and antineutrino beams. A detailed review on this issue is given in Ref.[44].

2. The sum of three neutrino masses ( $\sum m_i$ ) is always below the present cosmological experimental bound ( $\sum m_i < 0.23eV$ )[45, 46, 47, 48]. However, the next analysis[49] of Planck CMB satellite data in combination with more sensitive other cosmological and astrophysical experiments, such as Baryon oscillation spectroscopic survey(BOSS), The Dark energy survey(DES), The Large synoptic survey telescope(LSST) and the Euclid satellite, will bring

down the lower limit in a region of  $m_\nu \sim 0.1eV$  for inverted ordering and for normal mass ordering of neutrinos it will be pushed down to  $m_\nu \sim 0.05eV$ . Thus most of the present predicted range of  $\sum m_i$  ( $\sim 0.068 - 0.22$ ) $eV$  will be under scanner in the near future.

3. In the present work, the matrix element  $|m_{\nu_{ee}}|$ , which is constrained by the neutrinoless double beta decay ( $\beta\beta 0\nu$ ) experiment[50, 51, 52] varies within a range as shown in Table 9. EXO-200 experiment[53] has given a range on the upper limit of  $|m_{\nu_{ee}}|$  as  $|m_{\nu_{ee}}| < (0.14 - 0.38)eV$ . Thus the predicted values of the present work are below the above experimental value and are beyond the reach to be testified. However, it has been claimed that NEXT-100 experiment[54] will probe the value of  $|m_{\nu_{ee}}| \sim 0.1eV$ . We go optimistically with such findings in the near future.

We also provide a plot of Majorana phases in the right panel of Fig.1 and their allowed range is presented in Table 10. Determination of Majorana nature of neutrinos requires positive evidence from  $\beta\beta 0\nu$  experiment. However, in such process, CP symmetry is conserved, and, hence, to probe CP violating Majorana phases one has to look for the Lepton Flavor violating processes also[55].

Table 9: Allowed values of individual neutrino masses ( $m_i$ ) and their sum ( $\sum m_i$ ) and  $|m_{\nu_{ee}}|$

$m_1$ (eV)	$m_2$ (eV)	$m_3$ (eV)	$\sum m_i$ (eV)	$ m_{\nu_{ee}} $ (eV)
0.007-0.068	0.011-0.069	0.047-0.085	0.068-0.22	0.001-0.016

Table 10: Allowed values of Jarlskog measure ( $J_{CP}$ ), Dirac CP Phase ( $\delta_D$ ) and Majorana Phases ( $\alpha_M$  and  $\beta_M$ )

$J_{CP}$	$\delta_D(\text{deg.})$	$\alpha_M(\text{deg.})$	$\beta_M(\text{deg.})$
$(-0.041)-(0.041)$	$(-90)-(90)$	$(-90)-(90)$	$(-41)-(41)$

## 7 Summary and conclusion

Our goal of this work is to describe a phenomenologically viable effective light neutrino mass matrix ( $m_\nu$ ) with minimum number of parameters. The light neutrino mass matrix  $m_\nu$  is generated through

linear seesaw mechanism where along with standard  $SU(2)_L \times U(1)_Y$  particle contents, three right chiral singlet neutrinos ( $N_{R_i}$ ) and three other fermion singlets ( $S_{R_i}$ ) are present. The  $9 \times 9$  Majorana mass matrix obtained in this basis is further block diagonalized to get mass matrix for the light neutrinos. After imposing the assumption of absence of global  $U(1)_L$  symmetry breaking term we get the final working formula for  $m_\nu$  which is composed of three matrices  $m_D$ ,  $m_{DS}$  and  $m_{RS}$ . Without sacrificing any generality we are allowed to choose the charge lepton mass matrix and  $m_{RS}$  to be diagonal. To reduce the number of independent parameters our next idea is to invoke scaling ansatz in the two Dirac type matrices  $m_D$  and  $m_{DS}$ . The scaling ansatz is broken in final  $m_\nu$  by choice of different scale factors for  $m_D$  and  $m_{DS}$  to get rid of vanishing value of  $\theta_{13}$ .

The most important part of our analysis is to accommodate as many zeros as possible in those scaling ansatz invariant  $m_D$  and  $m_{DS}$ . It is noticed that we can get at most three 7 zero textures and nine 6 zero textures for both  $m_D$  and  $m_{DS}$ . So their combination give rise to  $12 \times 12 = 144$   $m_\nu$  matrices. Depending upon the positions of zeros and nonzero elements these 144 textures are generically represented by eight matrices denoted as  $t_i$  ( $i = 1, 2 \dots 8$ ) of which  $t_1$  and  $t_2$  are phenomenologically viable and the rest six are discarded. Among 144  $m_\nu$  matrices we get eighteen  $t_2$  type matrices and three  $t_1$  type matrices and the notable fact is that all three  $t_1$  type matrices are generated by the combination of 6 zero  $m_D$  and 6 zero  $m_{DS}$  where as  $t_2$  type matrices emerged in all type of combinations except those of 7 zero  $m_D$  with 7 zero  $m_{DS}$ .

All the  $t_2$  type matrices are recasted in two types of  $m_\nu$  (**Cat I** and **Cat II**) and  $t_1$  type matrices can be represented by single  $m_\nu$  matrix (**Cat III**) after phase redefinition and reparametrization. The numerical analysis is done thereafter. It is clear from the detailed numerical analysis that **Cat I** and **Cat II** matrices are disfavoured by oscillation data and the only surviving  $m_\nu$  belongs to **Cat III**. The mass ordering of the light neutrinos is normal and the value of  $\sum m_i$  is also below the present experimental lower limit. We conclude with a comment that to meet the phenomenological requirements we need at most two 6 zero matrices ( $m_D$  and  $m_{DS}$ ) and one diagonal matrix  $m_{RS}$  while working with linear seesaw mechanism. Increase in number of zeros in any of the two Dirac type matrices will make the resulting  $m_\nu$  phenomenologically invalid. Our numerical analysis of the survived texture, predicts quantitative nature of neutrino mass hierarchy and other observables, among them, except Majorana phases, all of them will be probed in the near future.

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